



Génie Electrique et Electronique  
Master Program  
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# EE-557 Semiconductor devices I

## Charge Transport

### Outline of the lecture

- Charge Transport
- thermal velocity
- Carrier transport: drift and diffusion

Read Chapter 4 of the reference book (on moodle)

#### References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (<http://ocw.mit.edu/>)

## Key questions

- Are carriers sitting still in thermal equilibrium?
- How do carriers move in an electric field?
- How do the energy band diagrams represent the presence of an electric field?
- How does a concentration gradient affect carriers?

## Carrier flow in semiconductors

Any **motion of free carriers** in a semiconductor leads to a **current**.

### Drift:

This motion can be caused by an **electric field** due to an externally applied voltage (electric field), since the carriers are charged particles. We will refer to this transport mechanism as **carrier drift**.

Ex: n-channel MOSFET

### Diffusion:

In addition, carriers also move from regions where the **carrier density is high to regions where the carrier density is low**. This carrier transport mechanism is due to the thermal energy and the associated random motion of the carriers. We will refer to this transport mechanism as **carrier diffusion**.

Ex: npn bipolar transistors

The **total current** in a semiconductor equals the **sum** of the **drift and the diffusion** currents.

What happens with electrons at a given temperature without electric field?

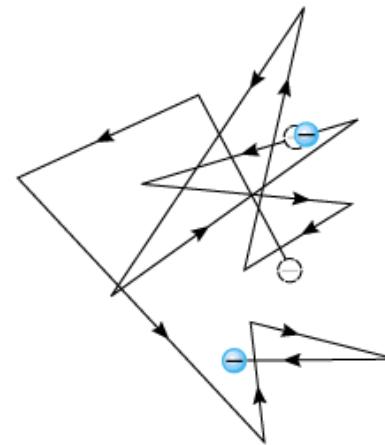
We can think of carriers as particles in **an ideal gas**.

At finite T, carriers have **finite thermal energy**. All this energy resides in the **kinetic energy** of the particles.

Carriers move in **random directions**: no net velocity, but average carrier velocity is thermal velocity:

$$v_{th} = \sqrt{\frac{8 kT}{\pi m_c^*}}$$

Zero electric field



$m_{ce}^* \equiv$  **conductivity effective mass** [ $eV \cdot s^2/cm^2$ ]

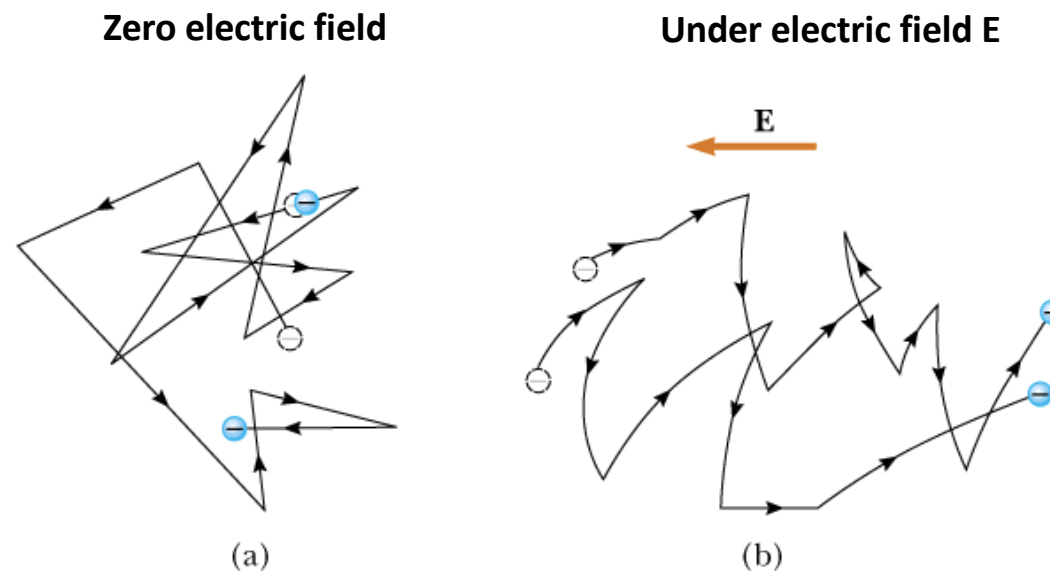
- This is the effective mass that connects the kinetic energy of an electrons with its velocity
- It accounts for all interactions between the carriers and the perfect periodic potential of the lattice.
- $m_{ce}^*$  and  $m_{ch}^*$  are different from the effective mass from the density of states (you can find the values in appendix B on moodle)

For electrons in Si at 300 K ( $m_{ce} = 0.28m_o$ ) and  $v_{the} = 2 \times 10^7$  cm/s

But... **semiconductor crystal is not perfect:**

- Atoms themselves are vibrating around their equilibrium position in the lattice
- There are impurities and crystal imperfections.

As carriers move around, they suffer frequent collisions: **scattering**



- **Mean free path**,  $l_{ce}$ : average distance travelled between collisions [cm].
- **Scattering time**,  $\tau_{ce}$ : average time between collisions [s].

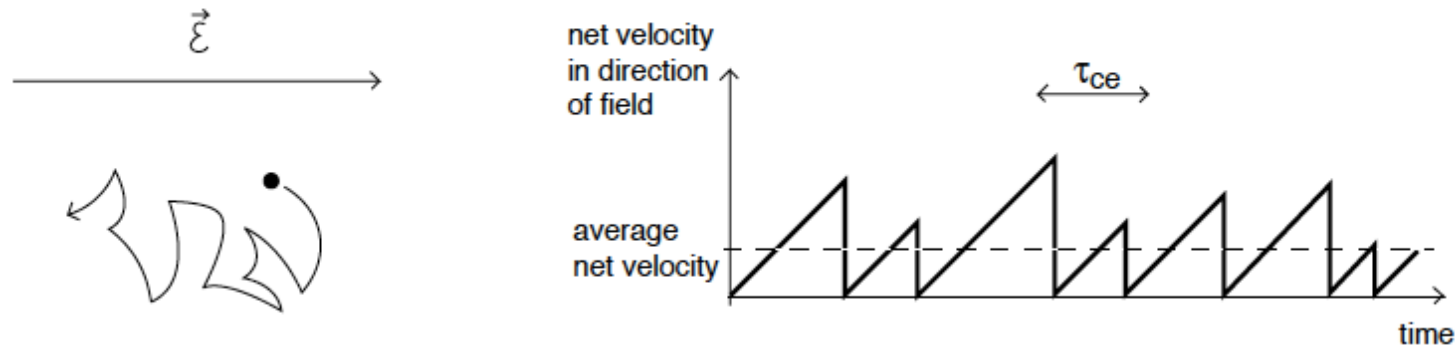
$$l_{CE} = \frac{\pi}{4} v_{th} \tau_{CE}$$

## Scattering mechanisms:

1. Lattice or **phonon scattering**: carriers collide with vibrating lattice atoms  
phonon absorption and emission  
⇒ some energy exchanged ( $\sim$  tens of meV )
2. **Ionized impurity scattering**: Coulombic interaction between charged impurities and carriers  
⇒ no energy exchanged
3. **Neutral impurity scattering** with neutral dopants, interstitials, vacancies, etc
4. **Surface scattering** in inversion layer (surface roughness)
5. **Carrier-carrier scattering (important when carrier concentration is high)**  
Order of magnitude of  $\tau_c < 1$  ps  
Then, order of magnitude of  $l_c < 50$  nm

## Drift

In the presence of an electric field, electrons drift:



Drift velocity

$$v_e^{drift} = -\frac{q\mathcal{E}\tau_{ce}}{m_{ce}^*}$$

$$v_e^{drift} = -\mu_e \mathcal{E}$$

$$\mu_e \equiv \text{electron mobility } [cm^2/V \cdot s]$$

**Electron mobility:** Corresponds to the **ease of carrier motion** in response to  $E$ . It depends on the **strength of the scattering mechanisms**.

In the presence of an electric field, electrons drift:

$$v_e^{drift} = -\mu_e \mathcal{E}$$

$$v_h^{drift} = \mu_h \mathcal{E}$$

**Mobility depends on:**

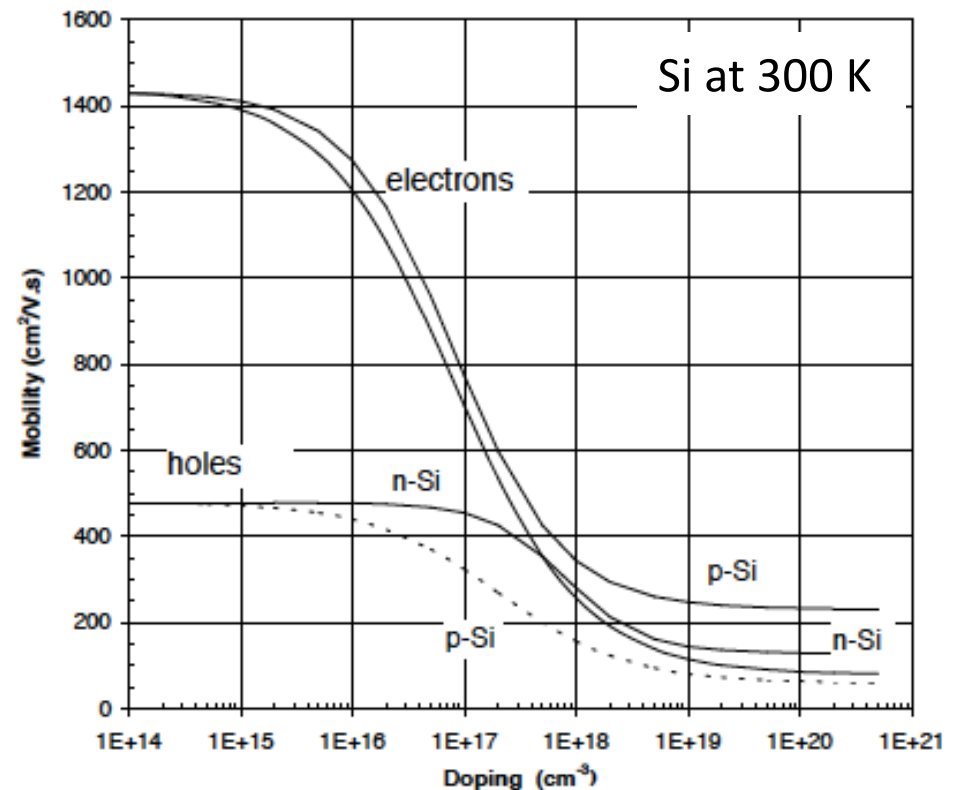
- **doping level**
- whether carrier is **majority** or **minority-type**.

at low  $n$ :

- Mobility is limited by phonon scattering
- thus independent of doping.

at high  $n$ :

- Mobility is limited by ionized impurity scattering;
- It is not a strong function of the type of dopant, but only on its concentration.
- Typically, attractive scattering is more pronounced than repulsive: majority carriers have lower mobility





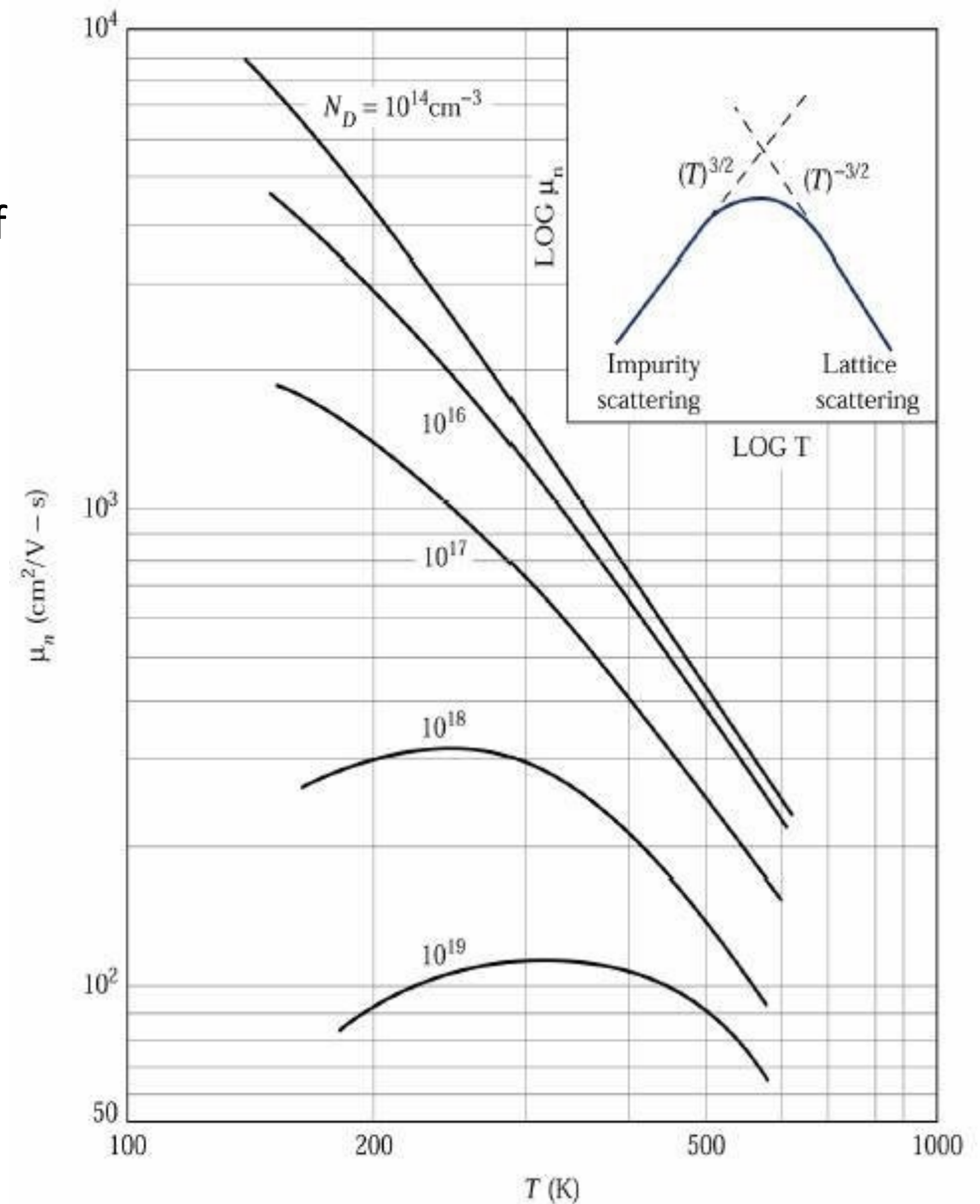
Increasing temperature and increasing doping results in reduction of mobility.

**Increasing temperature:** increases the number of phonons, which increases the probability that an electron will be scattered by a phonon.

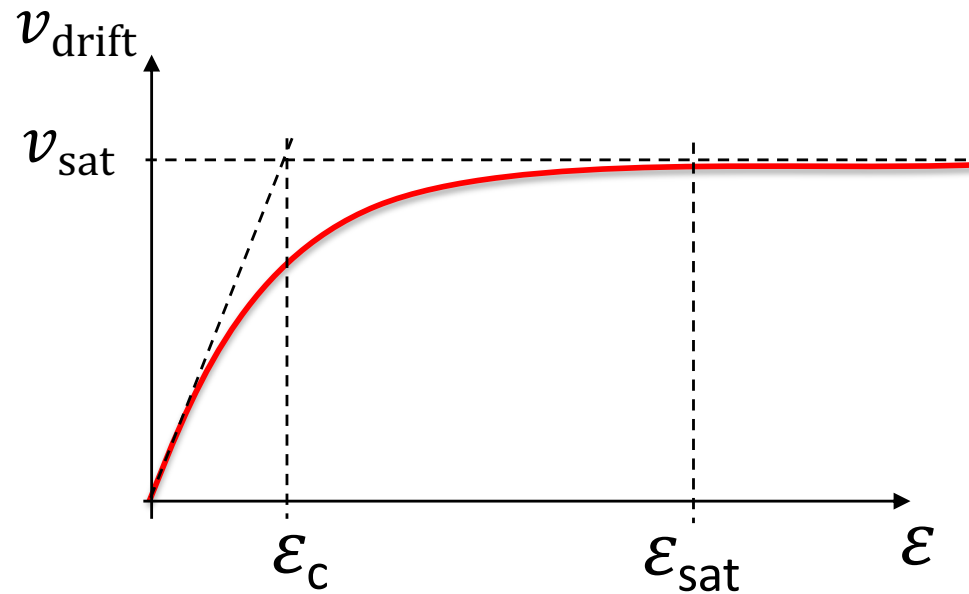
**Increasing doping:** each dopant atom can scatter electrons.

Thus:

higher doping level -> lower mobility  
higher temperature -> lower mobility



The **linear relationship** between drift velocity and electric field is **no longer valid at high fields**



Drift velocity vs. electric field fairly well described by:

$$v^{drift} = \mp \frac{\mu \mathcal{E}}{1 + \left| \frac{\mu \mathcal{E}}{v_{sat}} \right|}$$

Field required to saturate velocity:

$$\mathcal{E}_c = \frac{v_{sat}}{\mu}$$

$$\mathcal{E}_{sat} = 9\mathcal{E}_c$$

**Velocity saturation crucial in modern devices:**

if  $\mu = 500 \text{ cm}^2/\text{V.s}$ ,  $\mathcal{E}_{sat} = 2 \times 10^4 \text{ V/cm}$  (2V across  $1 \mu\text{m}$ )

Since  $\mu$  depends on doping,  $\mathcal{E}_{sat}$  depends on doping too.

The **linear relationship** between drift velocity and electric field **breaks at high fields**

Implicit assumption:

*quasi-equilibrium*, that is, scattering rates not much affected from equilibrium.

$$v^{drift} \sim \mathcal{E} \quad \text{only if} \quad v^{drift} \ll v_{th}$$

For **high  $E$** : carriers acquire substantial energy from  $E$

→ optical phonon emission strongly enhanced

scattering time:  $\sim 1/\mathcal{E}$

**Drift velocity saturates at**

$$v_{sat} \simeq \sqrt{\frac{8}{3\pi} \frac{E_{opt}}{m_c^*}}$$

**Mobility depends on doping level** and whether carrier is majority or minority-type.

For Si at 300 K:

- $v_{sat} \sim 10^7 \text{ cm/s}$  for electrons
- $v_{sat} \sim 6 \times 10^6 \text{ cm/s}$  for holes

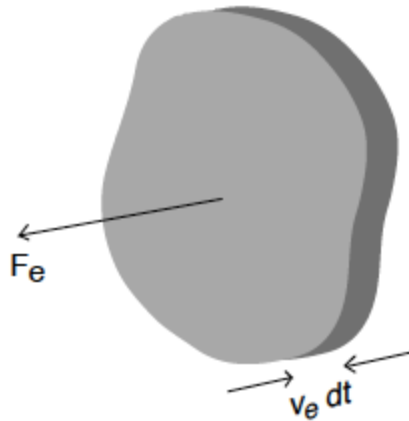
## Particle flux and current density

*particle flux*  $\equiv$  # particles crossing unity surface (normal to flow) per unit time [ $\text{cm}^{-2} \cdot \text{s}^{-1}$ ]

*current density*  $\equiv$  electrical charge crossing unity surface (normal to flow) per unit time [ $\text{C} \cdot \text{cm}^{-2} \cdot \text{s}^{-1}$ ]

$$J_e = -qF_e$$

*particle flux:*  $F_e = \frac{nv_e dt}{dt} = nv_e$



*Electron and hole current density:*

$$J_e = -qn v_e$$

$$J_h = qp v_h$$

## Drift current

Drift current (low fields):

$$J_e = q\mu_e n \mathcal{E}$$

$$J_h = q\mu_h p \mathcal{E}$$

**Total current:**

$$J = q(\mu_e n + \mu_h p) \mathcal{E}$$

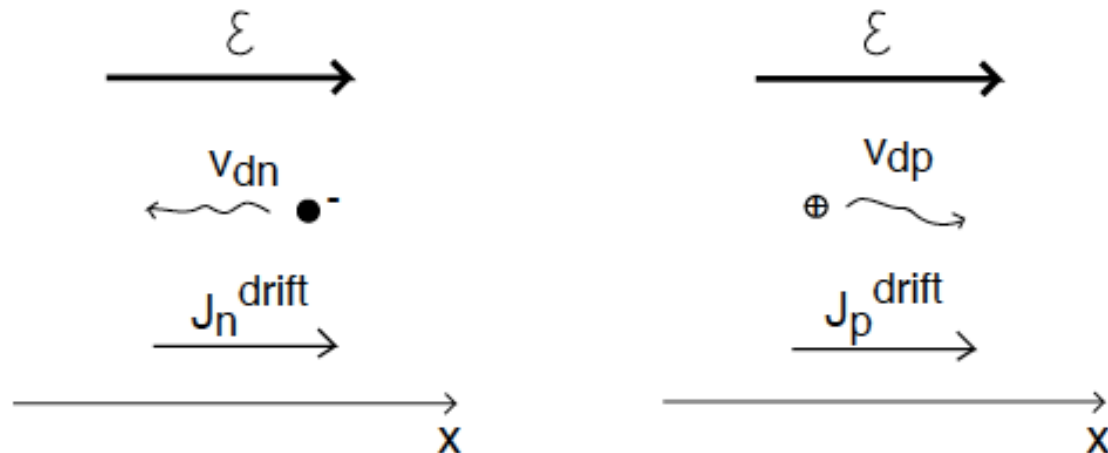
Ohm's law for semiconductors

**Electrical conductivity**  $[(\Omega \cdot \text{cm})^{-1}]$ :

$$\sigma = q(\mu_e n + \mu_h p)$$

**Electrical resistivity**  $[\Omega \cdot \text{cm}]$ :

$$\rho = \frac{1}{q(\mu_e n + \mu_h p)}$$



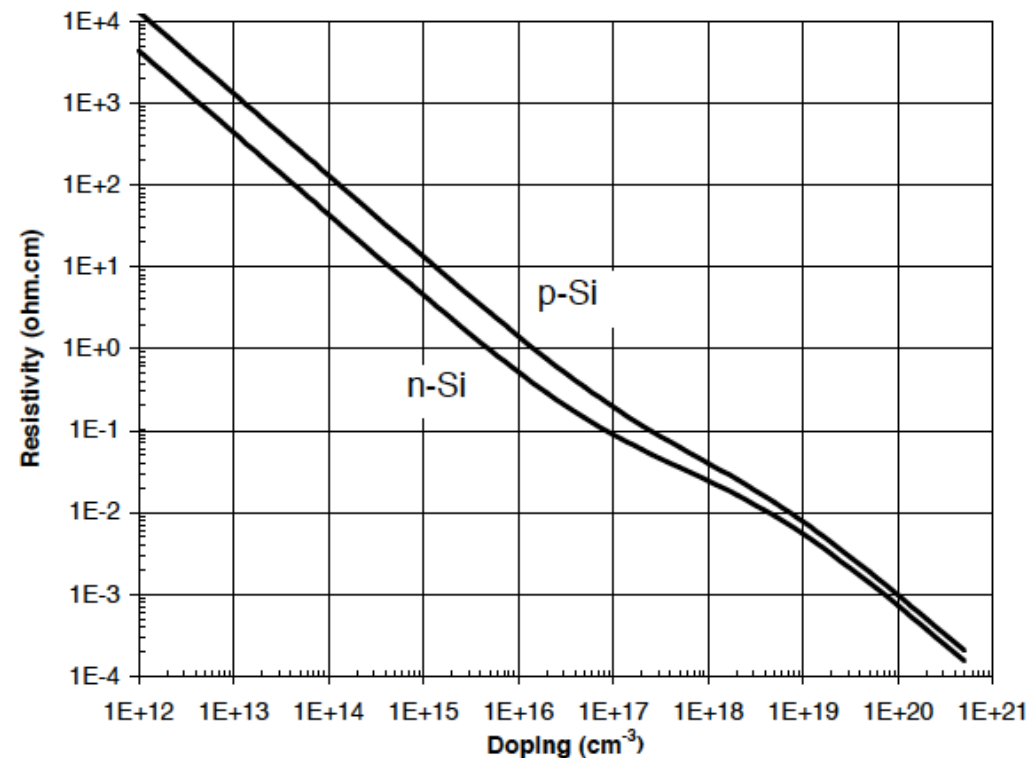
$\rho$  strong function of doping  $\Rightarrow$

frequently used by wafer vendors to specify doping level of substrates

-for n-type:  $\rho_n \simeq \frac{1}{q\mu_e N_D}$

-for p-type:  $\rho_p \simeq \frac{1}{q\mu_h N_A}$

Si at 300K:



Drift current (**high fields**):

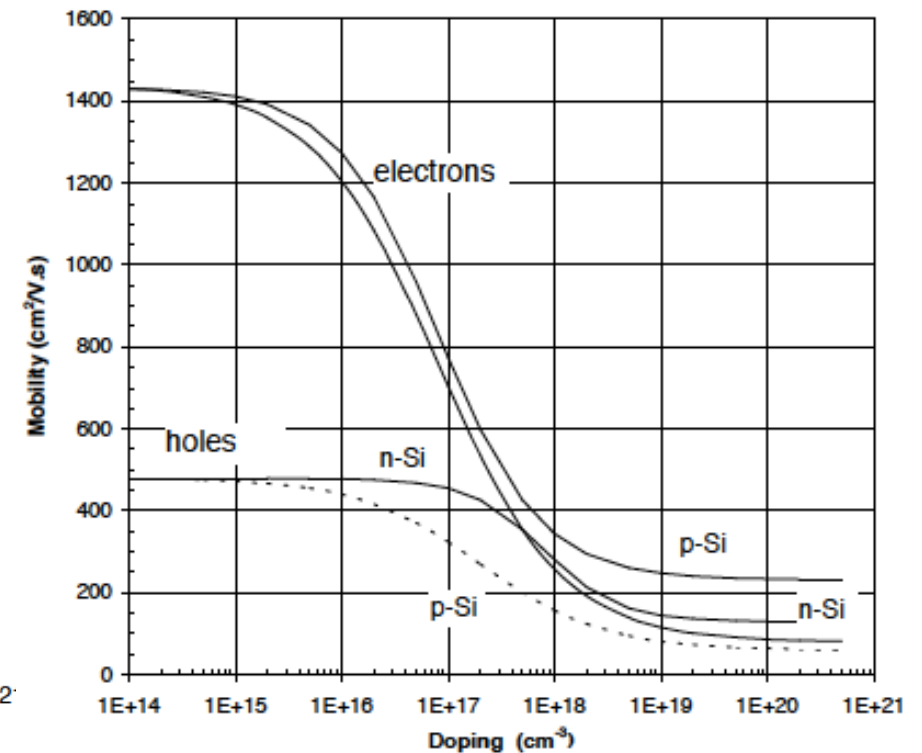
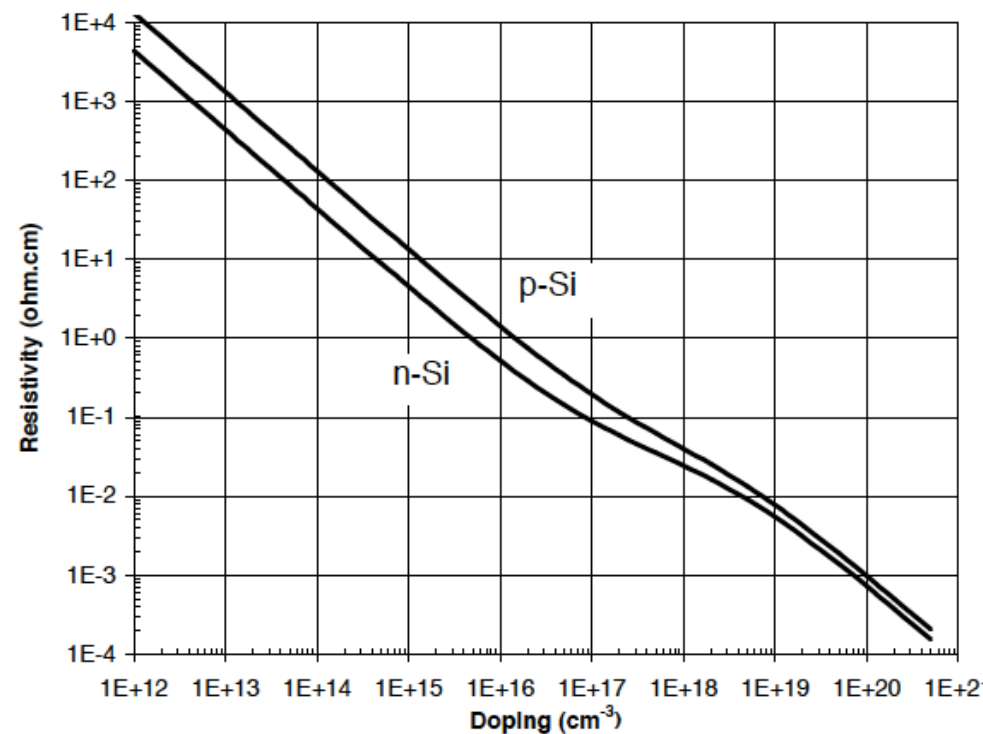
$$J_{sat} = qn v_{sat}$$

$$J_{hsat} = qp v_{hsat}$$

The only way to get **more current at high fields** is to **increase carrier concentration**.

Estimate the electrons and holes drift velocity, for a n-type Si with resistivity of 0.1 ohm cm and under an applied field of 1000 V/cm.

Si at 300K:





### *Sheet resistance*

The **sheet resistance** concept is used to characterize both wafers and thin doped layers, since it is typically **easier to measure than the resistivity** of the material.

The sheet resistance of a uniformly doped layer with resistivity,  $\rho$ , and thickness,  $t$ , is given by their ratio:

$$R_s = \frac{\rho}{t}$$

While the unit of the sheet resistance is Ohms, one refers to it as **Ohms per square**.

This nomenclature comes in handy when the resistance of a rectangular piece of material with **length, L**, and **width W** must be obtained. It equals the product of the sheet resistance and the number of squares or:

$$R = R_s \frac{L}{W}$$

## Transmission line method (TLM)

technique to determine the sheet resistance and contact resistance

The sheet resistance of a doped layer with resistivity,  $\rho$ , and thickness,  $t$ , is:

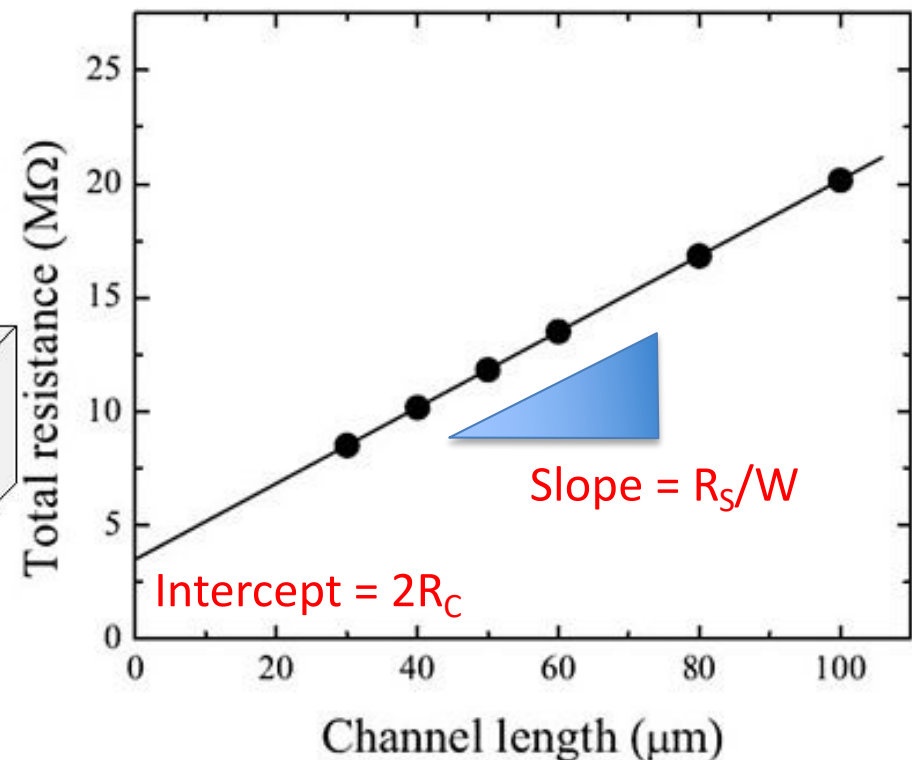
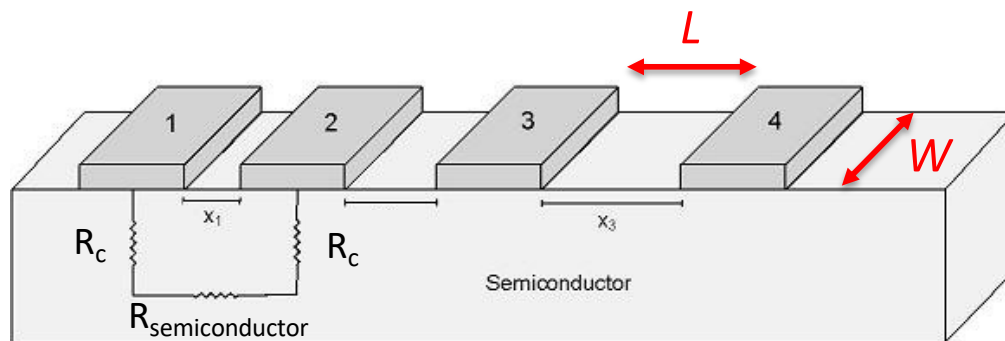
$$R_s = \frac{\rho}{t}$$

The resistance of a rectangular piece of material with length,  $L$ , and width  $W$

$$R = R_s \frac{L}{W}$$

Taking into account the contact resistance ( $R_c$ ):

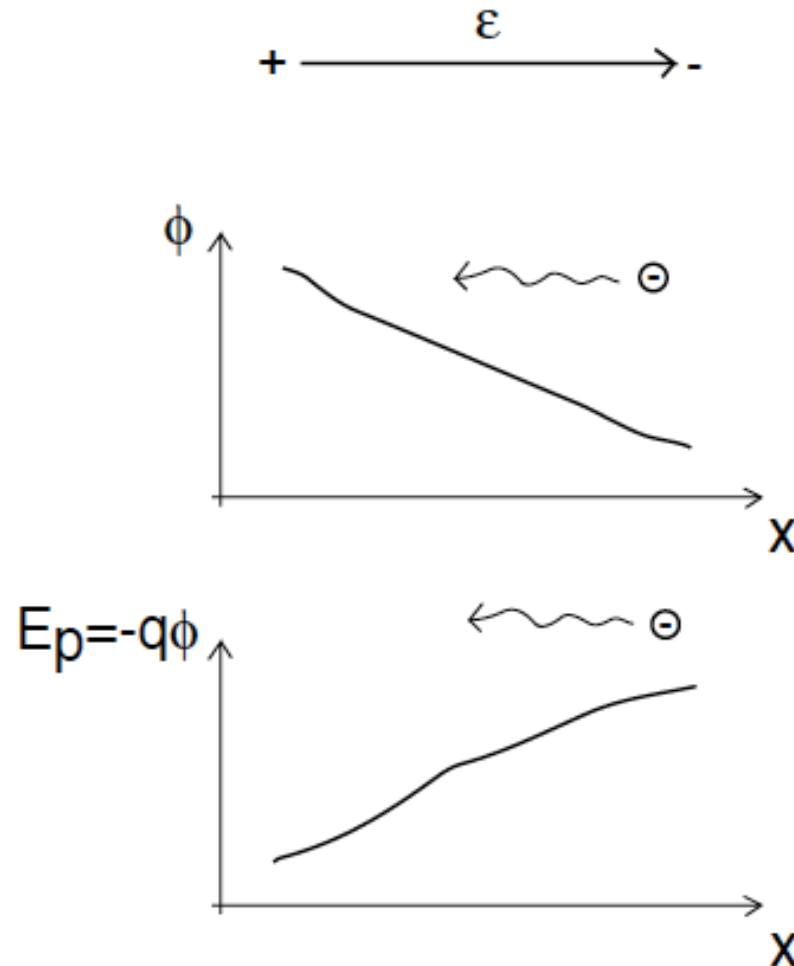
$$R = 2R_c + R_s \frac{L}{W}$$



## Energy band diagram under electric field

Energy band diagram under electric field:

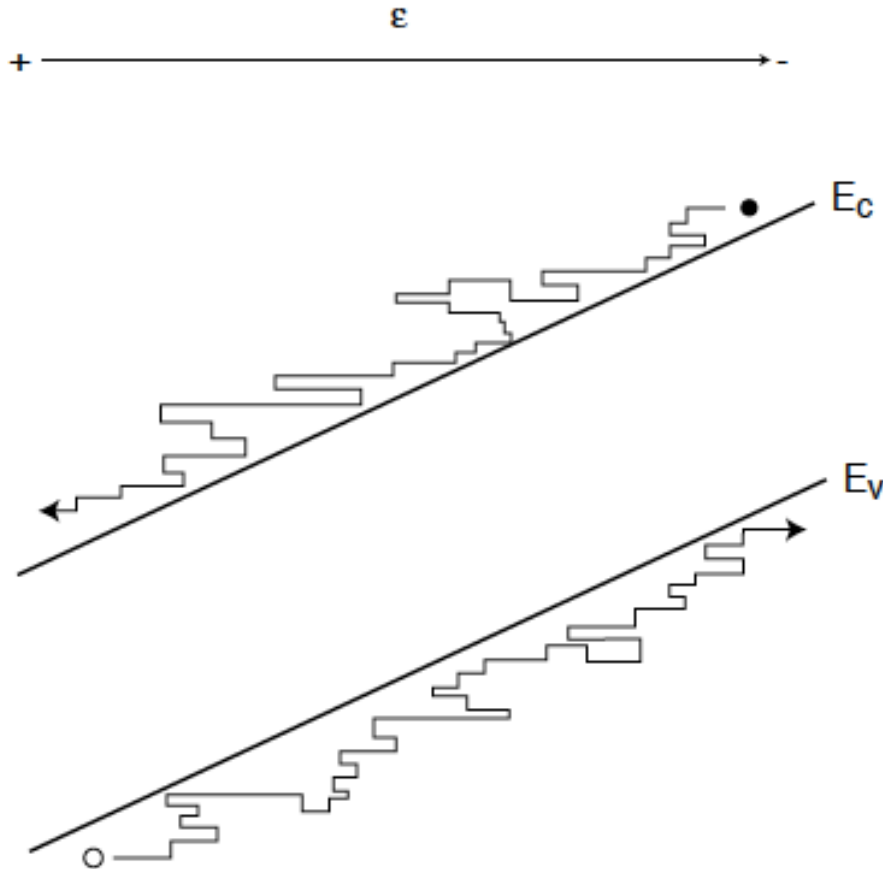
**Energy band diagram** needs to account for **potential energy of electric field**



Electron trades potential energy by kinetic energy as it moves to the left  
 $\rightarrow$  *total electron energy unchanged*

**Energy band diagram** is **picture of electron energy**

⇒ must add  $E_p$  to semiconductor energy band diagram ⇒ bands tilt: **Band bending**



Measuring from an arbitrary energy reference:

$$E_c + E_{ref} = E_p = -q\phi$$

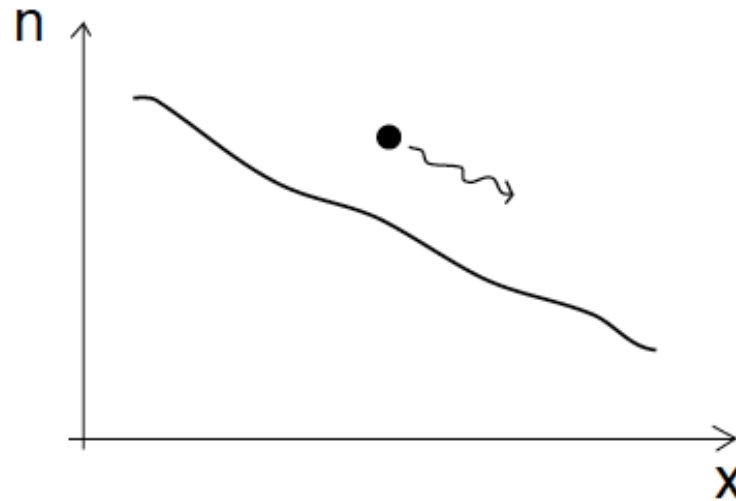
$$\mathcal{E} = -\frac{d\phi}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx}$$

**Shape of energy bands = shape of  $\phi$  with a minus sign.**

Can easily compute  $E$  from energy band diagram.

**Movement of particles** from regions of **high concentration** to regions of **low concentration**.

Diffusion produced by collisions with background medium (*i.e.*, vibrating Si lattice).



Diffusion flux is proportional to the gradient of concentration [Fick's first law]

$$F_e = -D_e \frac{dn}{dx}$$

$$F_h = -D_h \frac{dp}{dx}$$

$D \equiv$  diffusion coefficient [ $cm^2/s$ ]

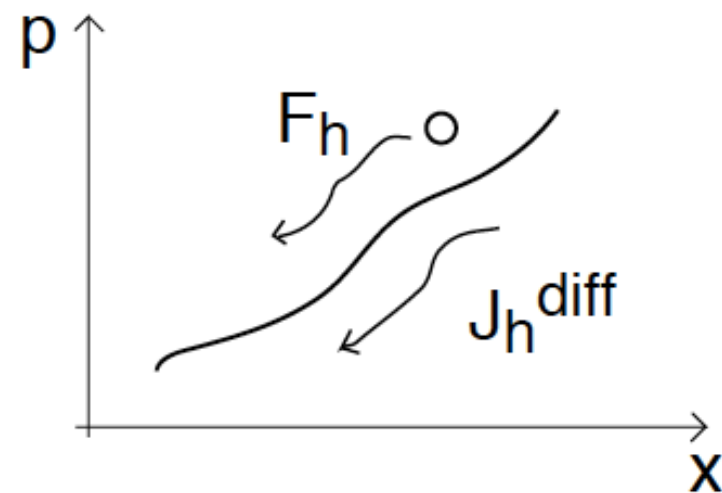
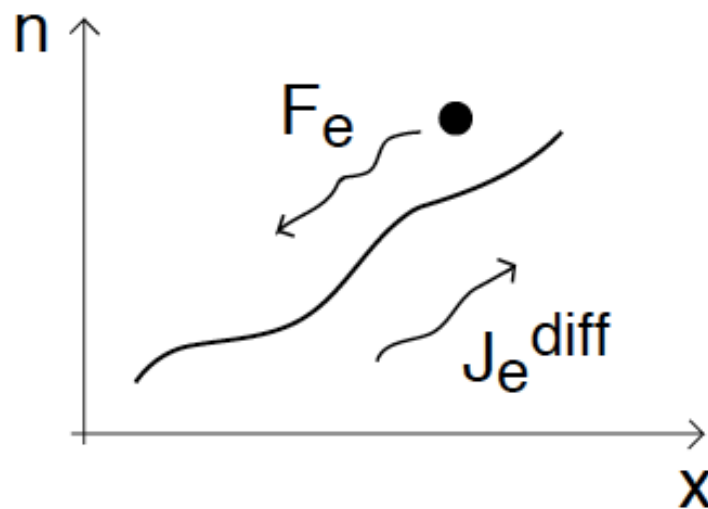
**Diffusion current:** Electrons and holes are charged particles and their diffusion creates current

$$F_e = -D_e \frac{dn}{dx}$$

$$F_h = -D_h \frac{dp}{dx}$$

$$J_e = qD_e \frac{dn}{dx}$$

$$J_h = -qD_h \frac{dp}{dx}$$



**In summary:**

$\mu$  relates to "ease" of carrier drift in an electric field.

$D$  relates to "ease" of carrier diffusion as a result of a concentration gradient.

**Is there a relationship between  $\mu$  and  $D$ ?**

**In summary:**

$\mu$  relates to "ease" of carrier drift in an electric field.

$D$  relates to "ease" of carrier diffusion as a result of a concentration gradient.

**Is there a relationship between  $\mu$  and  $D$ ?**

Yes, Einstein relation:

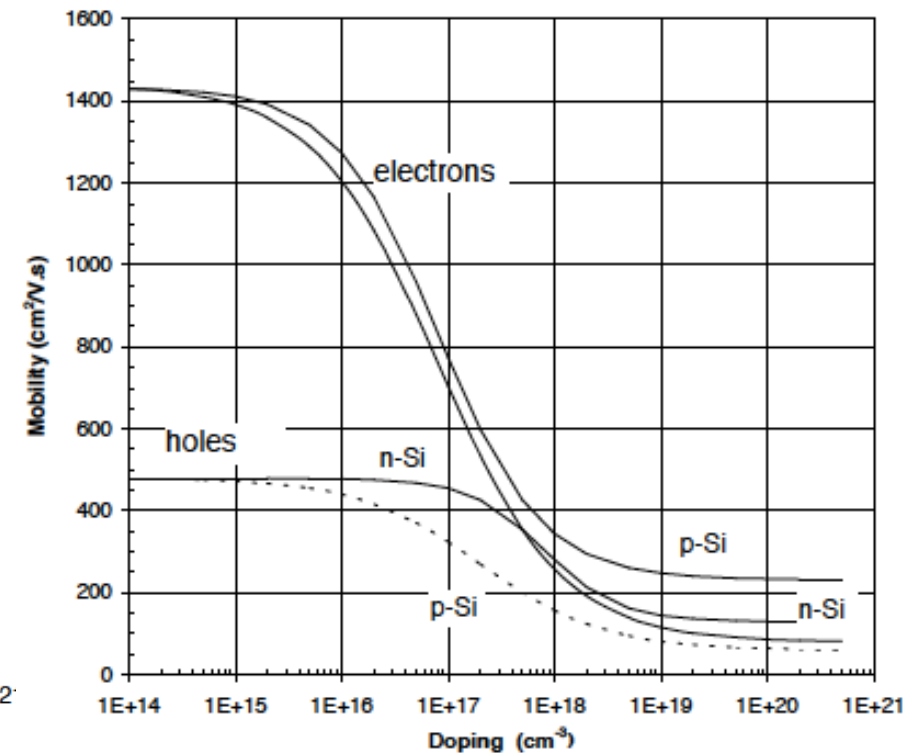
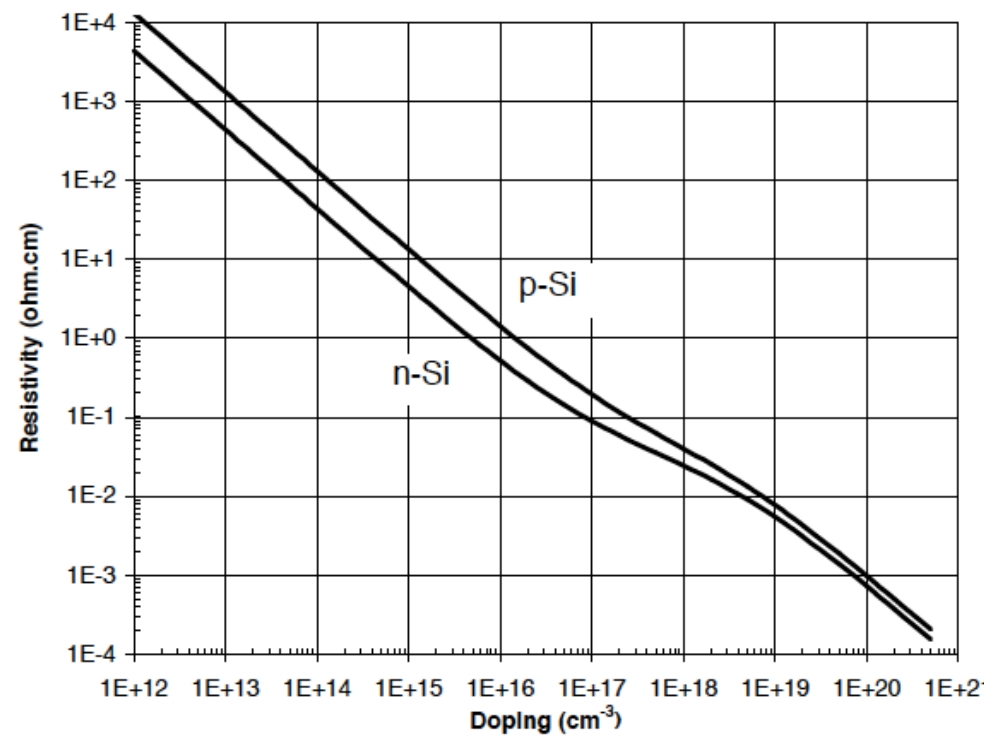
$$\frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{kT}{q}$$

Relationship between  $D$  and  $\mu$  only depends on  $T$ .



Estimate the diffusion coefficients for electrons and holes,  
for a p-type Si with resistivity of 0.1 ohm cm

Si at 300K:



At finite temperatures, carriers move around in a **random way** suffering many collisions: ***thermal motion***.

Dominant scattering mechanisms in bulk Si at 300K: **phonon scattering** and **ionized impurity scattering**.

Two processes for carrier flow in semiconductors: **drift and diffusion**.

General relationship between carrier net velocity (by drift or diffusion) and current density:

$$J_e = -qn v_e \qquad J_h = qp v_h$$

For low fields,  $v_{drift} \sim E$

For high fields,  $v_{drift} \sim v_{sat}$

Driving force for **diffusion**: **concentration gradient**.

**Order of magnitude of key parameters for Si at 300K:**

– electron mobility:  $\mu_e \sim 100 - 1400 \text{ cm}^2/V \cdot s$

– hole mobility:  $\mu_h \sim 50 - 500 \text{ cm}^2/V \cdot s$

– saturation velocity:  $v_{sat} \sim 10^7 \text{ cm/s}$

–  $v_{th} \sim 2 \times 10^7 \text{ cm/s}$

–  $\tau_c < 1 \text{ ps}$

–  $l_c < 50 \text{ nm}$